GRAPH THEORY

Tutorial – 11

1. Given an optimal coloring of a *k*-chromatic graph, prove that for each color *i* there is a vertex with color *i* that is adjacent to vertices of the other *k-1* colors.

2. Prove that if *G* is a color-critical graph, then the graph *G'* generated from it by applying Mycielski's construction is also color-critical.

3. Let G and H be k-critical graphs sharing only vertex v, with $vu \in E(G)$ and $vw \in E(H)$. Prove that, the graph $(G-vu) \cup (H-vw) \cup uw$ is k-critical.

4. Prove that, $X(C_n;k) = (k-1)^n+(-1)^n(k-1)$.

5. Let G be a maximal planar simple graph. Prove that, G^* is 2-edge connected and 3-regular.